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RE-ENTERING OF BODIES WITH A POSITIVE LIFT-TO-DRAG RATIO INTO THE EARTH'S ATMOSPHERE

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The problem of gliding descent of a smooth blunted body with a positive lift-to-drag ratio in the Earth's atmosphere is solved within the framework of the parabolized viscous shock layer model.

Introduction. When objects move in the atmosphere with high supersonic velocities, gas heating in the shock layer in a close vicinity of the body initiates various physicochemical processes, which should be taken into account to obtain a realistic physical flow pattern. In the present paper, numerical simulation of a supersonic, chemically nonequilibrium multispecies viscous gas flow is performed within the framework of the "parabolized" viscous shock layer model, which is a modification of the full viscous shock layer equations [1] and which was proposed initially for uniform gas flows [2, 3] and then for a multispecies mixture of gases [4]. Parabolized viscous shock layer equations, which are simplified full Navier–Stokes equations and contain all terms of boundary-layer equations and inviscid shock-layer equations in the hypersonic approximation, describe the entire disturbed region between the shock wave (its position is unknown *a priori*) and the body surface. The choice of this model is explained, first, by the fact that it is fairly accurate for smooth bodies within a rather wide vicinity of body bluntness, which is of practical interest and where the force and thermal loads are rather significant, and the shock layer itself remains thin [5]. Second, to solve the corresponding boundary-value problem within the framework of this model, one can use fast and economic marching methods, which is particularly important for three-dimensional flows. Finally, third, among all known modifications of the viscous shock layer equations, which retain the parabolic type, the parabolized viscous shock layer model allows a significant extension of the computational domain size in terms of the marching coordinate.

Formulation of the Problem. We consider the problem of a hypersonic flow of a chemically nonequilibrium mixture of gases around blunted bodies with a catalytic surface at angles of attack and sideslip. To solve the problem numerically, we introduce a curvilinear coordinate system x^i, which is normally attached to the body surface: the x^3 axis is directed along the normal to the body and the x^1 and x^2 axes are located on the body surface. The equations of the spatial "parabolized" viscous shock layer describe the flow between the body surface and the detached shock wave. Taking into account nonequilibrium chemical reactions and multispecies diffusion and ignoring thermal diffusion, diffusion thermal effect, and pressure diffusion, these equations in an arbitrary curvilinear coordinate system in dimensionless variables has the following form [4]:

$$D_\alpha(\rho u^\alpha \sqrt{g/g_{(\alpha\alpha)}}) + \sqrt{g} D_3(\rho u^3) = 0; \tag{1}$$

$$\rho(Du^\alpha + A^\alpha_{\beta\delta} u^\beta u^\delta) = -g^{\alpha\beta}\sqrt{g_{(\alpha\alpha)}}\, D_\beta P + D_3\Big(\frac{\mu}{\mathrm{Re}}\, D_3 u^\alpha\Big); \tag{2}$$

$$D_3 D_\alpha P = -D_\alpha(\rho A^3_{\beta\delta} u^\beta u^\delta); \tag{3}$$

$$\rho(Du^3 + A^3_{\beta\delta} u^\beta u^\delta) = -D_3 P; \tag{4}$$

$$\rho c_p DT - 2D^* P = D_3\Big(\frac{\mu c_p}{\sigma\,\mathrm{Re}}\, D_3 T\Big) + \frac{\mu}{\mathrm{Re}}\, B_{\alpha\beta} D_3 u^\alpha D_3 u^\beta - D_3 T \sum_{i=1}^{N} c_{pi} I_i - \sum_{i=1}^{N} h_i \dot{w}_i; \tag{5}$$

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$$\rho D c_i + D_3 I_i = \dot{w}_i, \qquad i = 1, \ldots, N - N_e; \tag{6}$$

$$\rho D c_i^* + D_3 I_i^* = 0, \qquad i = 1, \ldots, N_e - 1; \tag{7}$$

$$P = \rho T R \sum_{i=1}^{N} \frac{c_i}{m_i}; \tag{8}$$

$$\sum_{i=1}^{N_e} c_i^* = 1, \qquad \sum_{i=1}^{N_e} I_i^* = 0, \qquad c_p = \sum_{i=1}^{N} c_{pi} c_i. \tag{9}$$

Here $D_i \equiv \partial/\partial x^i$, $D^* \equiv (u^\alpha/\sqrt{g_{(\alpha\alpha)}})D_\alpha$, $D \equiv D^* + u^3 D_3$, $\mathrm{Re} = \rho_\infty V_\infty L/\mu(T_0)$, and $T_0 = 10^4$ K.

System (1)–(9) is closed by the Stefan–Maxwell relations

$$\sum_{j=1}^{N-1} a_{ij} I_j = -\frac{\mu}{\mathrm{Re}\,\mathrm{Sc}_{lN}} \sum_{j=1}^{N-1} b_{ij} D_3 c_j, \qquad i = 1, \ldots, N-1,$$

$$a_{ij} = -a_{ij}^* c_i, \qquad b_{ij} = -b_j^* c_i \qquad (i \neq j),$$

$$a_{ii} = \frac{\mathrm{Sc}_{iN}}{\mathrm{Sc}_{lN}} + \sum_{j=1,\ j\neq i}^{N-1} a_{ij}^* c_j, \qquad b_{ii} = 1 + \sum_{j=1,\, j\neq i}^{N-1} b_j^* c_j,$$

$$a_{ij}^* = \frac{m_N}{m_j}\,\frac{\mathrm{Sc}_{ij}}{\mathrm{Sc}_{lN}} - \frac{\mathrm{Sc}_{iN}}{\mathrm{Sc}_{lN}}, \qquad b_j^* = \frac{m_N}{m_j} - 1, \qquad \mathrm{Sc}_{ij} = \frac{\mu}{\rho D_{ij}}.$$

Here $V_\infty u^i$ are the physical components of the velocity vector along the corresponding coordinate axes, $\rho_\infty V_\infty^2 P$, $\rho_\infty \rho$, and $T_0 T$ are, respectively, the pressure, density, and temperature of the gas mixture consisting of N chemical components, $\mu(T_0)\mu$ is the viscosity, $(V_\infty^2/(2T_0))c_p$ is the specific heat capacity, σ is the Prandtl number, c_i, m_i, $0.5V_\infty^2 h_i$, $(V_\infty/(2T_0))c_{pi}$, $V_\infty \rho_\infty I_i$, and $V_\infty \rho_\infty \dot{w}_i/L$ are, respectively, the mass concentration, molecular weight, specific enthalpy, specific heat capacity, normal component of the diffuse flux vector, and the rate of formation of mass of the ith component as a result of chemical reactions, c_i^* and I_i^* are, respectively, the concentration and normal component of the diffuse flux vector of the ith chemical element ($i = 1, 2, \ldots, N_e$, where N_e is the number of elements), D_{ij} are the binary coefficients of diffusion, Sc_{ij} are Schmidt numbers, $R_G = V_\infty^2 R/T_0$ is the universal gas constant, $g_{\alpha\beta}$ and $g^{\alpha\beta}$ are, respectively, the covariant and contravariant components of the first quadratic form of the body surface ($g = g_{11}g_{22} - g_{12}^2$), and $A_{\beta\delta}^k$ are the known functions of the body shape [6]. Summation is performed over repeated indices, which are not enclosed in brackets. The Latin sub- and superscripts take the values 1, 2, or 3 (except for specially marked cases); the Greek sub- and superscripts are equal to 1 or 2. All linear dimensions are normalized to the characteristic linear size L. Hereinafter, the subscripts w, ∞, and s refer to quantities on the body surface, in the free stream, and behind the shock wave, respectively.

Boundary conditions on the shock wave and on the body surface are set for the differential equations (1)–(7). Generalized Rankine–Hugoniot equations in the hypersonic approximation with ignored chemical reactions inside the shock wave are used on the shock wave:

$$\begin{aligned} x^3 = x_s^3(x^1, x^2): \quad & \rho(u^3 - D^* x^3) = u_\infty^3, \\ & u_\infty^3(u^\alpha - u_\infty^\alpha) = \frac{\mu}{\mathrm{Re}} D_3 u^\alpha, \quad P = (u_\infty^3)^2, \\ & u_\infty^3(c_i - c_{i\infty}) + I_i = 0, \quad i = 1, \ldots, N - N_e, \\ & u_\infty^3(c_i^* - c_{i\infty}^*) + I_i^* = 0, \quad i = 1, \ldots, N_e - 1, \end{aligned} \tag{10}$$

$$\frac{\mu c_p}{\sigma u_\infty^3\,\mathrm{Re}}\, D_3 T = \sum_{i=1}^{N} c_{i\infty}(h_i - h_{i\infty}) - (u_\infty^3)^2 - B_{\alpha\beta}(u^\alpha - u_\infty^\alpha)(u^\beta - u_\infty^\beta).$$

The boundary conditions on the body surface, taking into account heterogeneous chemical reactions and ignoring heat removal inside the body, have the following form:

$$x^3 = 0: \quad u^i = 0, \quad q = \Gamma T^4,$$
$$q = \frac{\mu c_p}{\sigma \operatorname{Re}} D_3 T - \sum_{i=1}^{N} h_i I_i, \quad \Gamma = \frac{2\varepsilon_B \sigma_B T_0^4}{\rho_\infty V_\infty^3}, \tag{11}$$
$$I_i = \dot{r}_i, \quad i = 1, \ldots, N - N_e,$$
$$I_i^* = 0, \quad i = 1, \ldots, N_e - 1,$$

Here ε_B is the emissivity of the surface, σ_B is the Stefan–Boltzmann constant, and $\rho_\infty V_\infty \dot{r}_i$ is the rate of formation of the ith component due to heterogeneous reactions.

The dissociating air in the shock layer is represented as an ideal gas consisting of five chemical components: O_2, N_2, NO, O, and N, in which there proceed three dissociation-recombination reactions

$$O_2 + M \iff 2O + M,$$
$$N_2 + M \iff 2N + M,$$
$$NO + M \iff N + O + M$$

and three exchange reactions

$$N_2 + O_2 \iff 2NO,$$
$$NO + O \iff N + O_2,$$
$$NO + N \iff O + N_2$$

(M is a third element, which may be anyone of the five components).

The dependences of the rate constants of direct and reverse reactions on temperature were determined in accordance with [7]. The transfer coefficients and thermodynamic functions were calculated by the formulas given in [8–12].

The atmosphere is assumed to be isothermal with the density distribution ρ_∞ [g/cm^3] over the altitude H [km]: $\rho_\infty = 1.225 \cdot 10^{-3} \exp(-0.142H)$. It is assumed that heterogeneous catalytic reactions are first-order reactions $\dot{r}_i = -\rho k_{wi} c_i$ ($i \equiv$ O, N, NO), where $V_\infty k_{wi}$ is the rate constant of heterogeneous recombination. We consider two models of catalytic interaction of the gas and the solid surface. Model 1: $k_{wi} = 0$ (neutral surface); model 2: $k_{wi} = \infty$ (perfectly catalytic surface).

Method of Solution. We choose a polar coordinate system on the body surface; the origin is at the stagnation point [13]. At this point, the normal to the body surface coincides with the free-stream direction, which is determined by the angles of attack α and sideslip β. One family of coordinate lines consists of concentric "circles," and the other is a bunch of "beams" with the center in the origin.

Taking into account the special features of the chosen coordinate system on the surface, we pass to the new variables

$$\xi^\alpha = x^\alpha, \quad \zeta = \frac{1}{\Delta} \int_0^{x^3} \rho \, dx^3, \quad \Delta = \Delta(\xi^1, \xi^2) = \int_0^{x_s^3} \rho \, dx^3,$$

$$\frac{\partial f_\alpha}{\partial \zeta} = \frac{u^\alpha}{u_*^\alpha}, \quad \theta = \frac{T}{T_*}, \quad u_*^1 = u_\infty^1, \quad u_*^2 = \xi^1, \quad T_* = \frac{1}{2}(u_\infty^3)^2,$$

$$\rho u^3 \sqrt{g} = -\frac{\partial}{\partial \xi^\alpha}(\psi_*^{(\alpha)} f_\alpha) - \psi_*^{(\alpha)} \frac{\partial f_\alpha}{\partial \zeta} \frac{\partial \zeta}{\partial x^\alpha}, \quad \psi_*^\alpha = \Delta u_*^\alpha \sqrt{g/g_{(\alpha\alpha)}}, \quad P_\alpha = \frac{1}{(\xi^1)^\alpha} \frac{\partial P}{\partial \xi^\alpha},$$

$$X_i = I_i/\Delta, \quad i = 1, \ldots, N - N_e, \qquad X_i^* = I_i^*/\Delta, \quad i = 1, \ldots, N_e.$$

As a result, it becomes possible to resolve the singularities in the stagnation point.

In the new variables, the continuity equation (1) is identically satisfied; the whole system of differential equations (2)–(9) with the boundary conditions (10) and (11) in these variables is given in [4].

The Stefan–Maxwell equations are written in the form [14]

$$X_i = \alpha_i \frac{\partial c_i}{\partial \zeta} + \beta_i c_i, \qquad i = 1, \ldots, N - 1. \tag{12}$$

To determine the flux X_i, we integrate the corresponding continuity equation for the ith component; to calculate the concentration of this component c_i, we use this equation with the substitution of X_i from Eq. (12). This approach allows finding the unknown concentrations of gases of the mixture without prior resolution of the Stefan–Maxwell relations relative to diffuse fluxes.

Thus, the resultant system, which described the mixture flow in the shock layer, contains two third-order equations for the transverse coordinate ζ relative to the stream functions f_α, a second-order equation for the temperature θ, a second-order equation for the coordinate x^3 (which is a corollary of the momentum equation in projection onto the normal to the body surface, the continuity equation, and the equation of state), two first-order equations for the longitudinal components of the pressure gradient P_α, $N-1$ first-order equations for diffuse fluxes X_i, and $N-1$ second-order equations for the concentrations c_i.

The normal component of the velocity vector u^3 and the density ρ are determined by the formulas

$$u^3 = -A\,\frac{\partial x^3}{\partial \zeta} + (\xi^1)^{2-\alpha}\,\frac{\partial f_\alpha}{\partial \zeta}\,\frac{\partial x^3}{\partial \xi^\alpha}, \quad \rho = \frac{\Delta\xi^1}{\sqrt{g}}\left(\frac{\partial x^3}{\partial \zeta}\right)^{-1}.$$

For numerical integration of the second- and third-order differential equations relative to the transverse coordinate ζ, we used an implicit finite-difference scheme with an order of approximation $O(\delta\xi^1)+O(\delta\xi^2)^2+O(\delta\zeta)^4$, which is an extension of the scheme of [15] to a three-dimensional case and employs a variable step along the coordinate ζ. In the convective operator, the derivatives with respect to the marching coordinate ξ^1 are replaced by upstream differences, and the derivatives with respect to the circular coordinate ξ^2 are approximated by central differences on the basis of the solution obtained at the previous iteration on the current "circle" $\xi^1 + \delta\xi^1 = \text{const}$.

The first-order equations are integrated by Simpson's method with a fourth-order accuracy in terms of ζ.

The value of $\Delta(\xi^1, \xi^2)$ is determined after completing the calculation of the whole "circle" at each iteration by the method of cylindrical sweeping [16]; thus, it is a linking function for solutions obtained at all computational points of the current "circle."

Since the coordinate system introduced on the body surface (ξ^1, ξ^2) degenerates in the stagnation point, we used a nondegenerate curvilinear coordinate system to solve the initial equations at this point. The resultant solution was recalculated to the coordinate system (ξ^1, ξ^2, ζ) using the algebraic formulas of [13] and was assumed to be the initial solution for the entire computational domain.

The special feature of the method proposed is that its implementation does not require planes of symmetry in the flow (therefore, it is possible to perform calculations for the general case of the flow around bodies at angles of attack and sideslip), and numerical differentiation of velocity and temperature profiles across the shock layer is not needed in calculating the friction and heat-transfer coefficients on the body surface.

The calculations show that this method is stable and economical and allows one to obtain a solution of the system of parabolized viscous shock layer equations within a wide range of the governing parameters of the problem.

Results. The re-entry body was assumed to be an elliptic paraboloid, whose equation in the Cartesian coordinate system has the form $2z = (x/b)^2 + (y/c)^2$, where $b = 1$ and $c = 1.5811$. The ratio of the major curvatures at the apex of this paraboloid is $k = 0.4$. The characteristic linear scale in the problem was chosen as the least radius of the major curvatures at this point $L = 0.5$ m. The emissivity of the surface was $\varepsilon_B = 0.85$.

To verify the reliability of the results given below, we solved the problem of the flow around this paraboloid under conditions of its motion along the re-entry trajectory to the Earth's atmosphere; the paraboloid centerline coincides with the free-stream direction. The values of the temperature of the heat-insulated surface at the paraboloid apex, which were obtained in the present work, coincide with the data of [17] for both models of heterogeneous catalytic reactions.

The trajectory considered in the present paper (Table 1) was borrowed from [18]. The computational domain is a space enclosed between the shock-wave and paraboloid surfaces and the surface formed by the normals to the paraboloid cross section by the plane $z = 0.8$. This plane was chosen so that the stagnation point was inside the computational domain along the whole trajectory (i.e., for given α and β). Thus, the computation of the flow of a multispecies mixture near the surface of the elliptical paraboloid in the chosen curvilinear coordinate system starts from the stagnation point with the coordinate $\xi^1 = 0$ and is terminated in a given cross section, which coincides with the coordinate line $\xi^1 = 1$.

Figure 1 shows the calculation results for the surface temperature at the stagnation point for the paraboloid moving along the trajectory for different cases of proceeding of heterogeneous catalytic reactions (curves 1 correspond

TABLE 1

H, km	t, sec	V, m/sec	α, deg	β, deg
122.0	0	7810	34	30
99.7	190	7840	34	24
76.1	430	7680	34	70
65.3	900	6240	32	53
48.4	1470	2724	30	10
46.8	1525	2440	30	17
27.4	1790	985	15	29
21.8	1905	463	10	−10
21.4	1909	454	10	−10

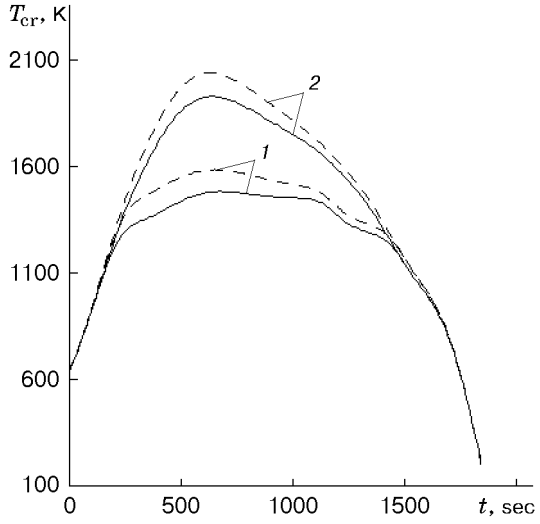

Fig. 1

TABLE 2

H, km	t, sec	T_{cr}, K		T_{max}, K		T_a, K	
		Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
98.6	200	1215	1250	1228	1257	1170	1188
78.3	400	1385	1720	1417	1743	1369	1646
68.7	600	1477	1927	1509	1959	1464	1880
67.9	640	1483	1931	1516	1964	1468	1884
66.3	800	1474	1876	1500	1903	1430	1800
63.3	1000	1456	1755	1477	1770	1421	1691
57.0	1200	1372	1587	1385	1599	1347	1537
50.3	1400	1265	1314	1280	1324	1250	1280

to a perfectly catalytic surface and curves 2 refer to a neutral surface). We considered the trajectory descent of the body with taking into account its lift-to-drag ratio (solid curves) and ignoring the latter ($\alpha = \beta = 0$) (dashed curves). In the latter case, the stagnation point coincides with the paraboloid apex. For realistic values of the angles of attack and sideslip, the temperature at the stagnation point is lower; the maximum decrease in temperature reaches 100–110 K, depending on the model of heterogeneous catalytic reactions. Since the stagnation point is not the point of the maximum surface temperature for nonzero angles of attack (in contrast to the case $\alpha = 0$), it cannot be considered as the characteristic point in this aspect.

Table 2 gives the temperature at the stagnation point T_{cr}, in the most heat-loaded point of the surface T_{max}, and at the paraboloid apex T_a at some points of the trajectory for a neutral surface (model 1) and a perfectly catalytic surface (model 2). In both numerical models, the maximum of characteristic temperatures is reached at an altitude of approximately 68 km. (The greatest temperature at the nose tip of the fuselage, which is approximately equal to 1920 K, is given in [18].)

For design purposes, of interest is the mean-integral heat flux over the surface, which is calculated by the formula

$$q_S = \frac{1}{S}\int q\,dS = \int_0^{2\pi}\int_0^1 q(\xi^1,\xi^2)\sqrt{g(\xi^1,\xi^2)}\,d\xi^1\,d\xi^2\left(\int_0^{2\pi}\int_0^1 \sqrt{g(\xi^1,\xi^2)}\,d\xi^1\,d\xi^2\right)^{-1},$$

where S is the body-surface area.

Figure 2 shows the heat flux at the stagnation point q_{cr} (curves 1) and points q_S (curves 2) as a function of flight time for a perfectly catalytic (solid curves) and neutral (dashed curves) surfaces. The points in Fig. 2 indicate the values $q = Kq_{cr}$, where $K = 0.527$ for a perfectly catalytic surface and 0.556 for a neutral surface. The

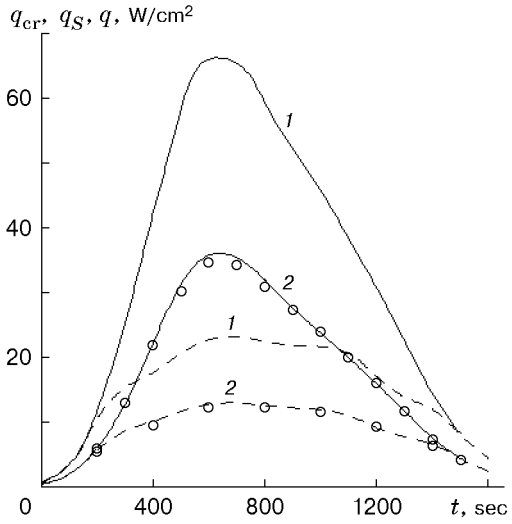

Fig. 2

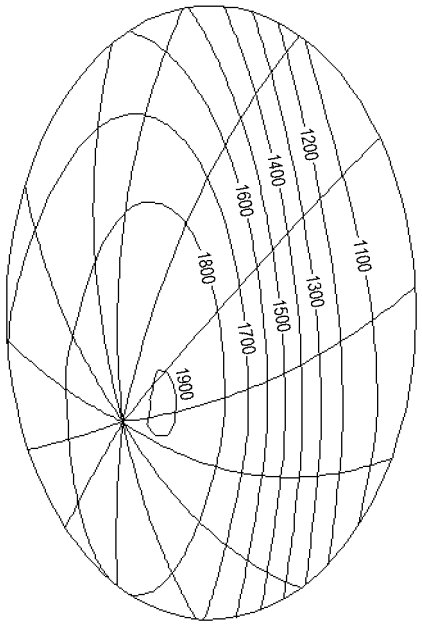

Fig. 3

difference between this linear dependence of q and the distribution of q_S is less than 4%. The reason is that the heat flux to the body surface, normalized to its value at the stagnation point, depends weakly on the governing parameters of the problem for trajectory motion, as was noted in [2–5, 13]. It is seen in Fig. 2 that the positions of the maximum coincide for all distributions ($H \approx 68$ km).

Figure 3 gives an idea of the character of heating of the surface and location of heat-loaded regions on the latter. Figure 3 shows the projections of temperature isolines of a perfectly catalytic surface of the paraboloid onto the plane of Cartesian coordinates (x, y) for $t = 800$ sec. The diverging "beams" are coordinate lines $\xi^2 = \text{const}$; the center of this bunch is the stagnation point. There are only two coordinate lines $\xi^1 = \text{const}$ in Fig. 3: the stagnation point ($\xi^1 = 0$) and the boundary of the computational domain ($\xi^1 = 1$). The paraboloid apex is in the center of the ellipse. The temperature distribution along the body surface at an arbitrary time of descent along the trajectory considered has a complex and significantly asymmetric character. The zone of elevated temperatures moves together with the stagnation point, though the latter is not the point of the maximum of these distributions. The point of the maximum is shifted toward the maximum mean curvature of the body surface.

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REFERENCES

1. R. T. Davis, "Numerical solution of the hypersonic viscous shock layer equations," *AIAA J.*, **8**, No. 5, 843–851 (1970).
2. A. I. Borodin and S. V. Peigin, "Spatial flow around blunted bodies within the framework of the parabolized viscous shock layer model," *Mat. Model.*, **5**, No. 1, 16–25 (1993).
3. A. I. Borodin and S. V. Peigin, "Parabolized viscous shock layer model for studying spatial hypersonic viscous gas flow around bodies," *Teplofiz. Vys. Temp.*, **31**, No. 6, 925–933 (1993).
4. A. I. Borodin, V. Yu. Kazakov, and S. V. Peigin, "Simulation of multispecies, chemically nonequilibrium flows within the framework of the parabolized spatial viscous shock layer model," *Mat. Model.*, **8**, No. 10, 3–14 (1996).
5. A. I. Borodin and S. V. Peigin, "Investigation of spatial viscous gas flows within the framework of parabolic flow models," *Teplofiz. Vys. Temp.*, **34**, No. 3, 429–435 (1996).
6. Yu. D. Shevelev, *Three-Dimensional Problems of the Laminar Boundary Layer Theory* [in Russian], Nauka, Moscow (1977).
7. E. W. Miner and C. H. Lewis, "Hypersonic ionizing air viscous shock-layer flows over nonanalytic blunt bodies," NASA Report No. 2550, Washington (1975).

8. C. R. Wilke, "A viscosity equation of gas mixtures," *J. Chem. Phys.*, **18**, No. 4, 517–519 (1959).
9. E. A. Mason and S. C. Saxena, "Approximate formula for the thermal conductivity of gas mixtures," *Phys. Fluids*, **1**, No. 5, 361–369 (1958).
10. R. A. Svehla, "Estimated viscosities and thermal conductivities of gases at high temperatures," NASA Tech. Report No. R-132, Washington (1962).
11. J. O. Hirchfelder, C. Curtiss, and R. Bird, *Molecular Theory of Gases and Liquids*, John Wiley and Sons, New York (1954).
12. L. V. Gurvich, I. V. Veits, V. A. Medvedev, et al., *Thermodynamic Properties of Individual Substances: Handbook* [in Russian], Nauka, Moscow (1978).
13. A. I. Borodin and S. V. Peigin, "Spatial thin viscous shock layer in the absence of planes of symmetry in the flow," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 2, 150–158 (1989).
14. É. A. Gershbein, "Laminar multispecies boundary layer under intense injection," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 1, 64–73 (1970).
15. I. V. Petukhov, "Numerical calculation of two-dimensional flows in the boundary layer," in: *Numerical Methods for Solving Differential and Integral Equations and Quadrature Formulas* [in Russian], Nauka, Moscow (1964), pp. 305–325.
16. A. A. Samarskii and E. S. Nikolaev, *Methods of Solving Grid Equations* [in Russian], Nauka, Moscow (1978).
17. É. A. Gershbein, V. G. Krupa, and V. S. Shchelin, "Spatial, chemically nonequilibrium viscous shock layer on a catalytic surface, taking into account contingent heat transfer," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 6, 140–146 (1985).
18. *Astronautics and Rocket Dynamics*, Digest of VINITI, No. 34 (1974).